Space-Time Compactification, Non-Singular Black Holes, Wormholes and Braneworlds via Lightlike Branes*

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Abstract

We describe a concise general scheme for constructing solutions of Einstein-Maxwell-Kalb-Ramond gravity-matter system in bulk space-time interacting self-consistently with one or more (widely separated) codimension-one electrically charged *lightlike* branes. The lightlike brane dynamics is explicitly given by manifestly reparametrization invariant world-volume actions. We present several explicit classes of solutions with different physical interpretation as wormhole-like space-times with one, two or more "throats", singularity-free black holes, brane worlds and space-times undergoing a sequence of spontaneous compactification-decompactification transitions.

1. Introduction

Lightlike branes (*LL-branes* for short) are singular null (lightlike) hypersurfaces in Riemannian space-time which provide dynamical description of various physically important phenomena in cosmology and astrophysics such as: (i) impulsive lightlike signals arising in cataclysmic astrophysical events (supernovae, neutron star collisions) [1]; (ii) dynamics of horizons in black hole physics – the so called "membrane paradigm" [2]; (iii) the thin-wall approach to domain walls coupled to gravity [3, 4, 5].

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More recently, the relevance of *LL-branes* in the context of non-perturbative string theory has also been recognized [6].

Starting with the pioneering papers [3, 4, 5] the *LL-branes* have been exclusively treated in a "phenomenological" manner, *i.e.*, without specifying an underlying Lagrangian dynamics from which they may originate. On the other hand, in the last few years we have proposed in a series of papers [7, 8, 9, 10] a new class of concise manifestly reparametrization invariant world-volume Lagrangian actions, providing a derivation from first principles of the *LL-brane* dynamics. The following characteristic features of the new *LL-branes* drastically distinguish them from ordinary Nambu-Goto branes:

(a) They describe intrinsically lightlike modes, whereas Nambu-Goto branes describe massive ones.

(b) The tension of the *LL-brane* arises as an *additional degree of freedom*, whereas Nambu-Goto brane tension is a given *ad hoc* constant. The latter characteristic feature significantly distinguishes our *LL-brane* models from the previously proposed *tensionless* p-branes (for a review, see Ref.[11]). The latter rather resemble p-dimensional continuous distributions of independent massless point-particles without cohesion among the latter.

(c) Consistency of *LL-brane* dynamics in a spherically or axially symmetric gravitational background of codimension one requires the presence of an event horizon which is automatically occupied by the *LL-brane* ("horizon straddling" according to the terminology of Ref.[4]).

(d) When the *LL-brane* moves as a *test* brane in spherically or axially symmetric gravitational backgrounds its dynamical tension exhibits exponential "inflation/deflation" time behavior [8] – an effect similar to the "mass inflation" effect around black hole horizons [12].

An intriguing novel application of *LL-branes* as natural self-consistent gravitational sources for *wormhole* space-times has been developed in a series of recent papers [9, 10, 13, 14]. In what follows, when discussing wormholes we will have in mind precisely this physically important class of "thin-shell" traversable Lorentzian wormholes first introduced by Visser [15, 16]. For a comprehensive general review of wormhole space-times, we refer to [16, 17].

In the present work we describe a concise systematic scheme for constructing solutions of Einstein-Maxwell-Kalb-Ramond gravity-matter system in bulk space-time coupled self-consistently to one or more (widely separated) codimension-one electrically charged *LL-branes*. The solutions describe bulk space-time manifolds consisting of several space-time regions ("universes") with different (in general) geometries such that: (i) each separate "universe" is a "vacuum" solution of Einstein-Maxwell-Kalb-Ramond equations (*i.e.*, without the presence of *LL-branes*); (ii) the separate "universes" are pairwise matched (glued together) along some of their common horizons; (iii) each of these common matching horizons is automatically occupied by one *LL-brane* ("horizon straddling") which generates space-time varying cosmological constants in the various matching "universes".

We present several explicit types of solutions with different physical interpretation such as: (a) wormhole-like space-times with one, two or more "throats"; (b) non-singular black holes; (c) brane worlds; (d) space-times undergoing a sequence of spontaneous compactification/decompactification transitions triggered by *LL-branes*.

2. Lagrangian Formulation of Lightlike Brane Dynamics

In a series of previous papers [7, 8, 9, 10, 13, 18] we have proposed manifestly reparametrization invariant world-volume Lagrangian formulation of *LL-branes* in several dynamically equivalent forms. Here we will use the Nambu-Goto-type formulation given by the world-volume action:

$$S_{\rm LL} = -\int d^{p+1}\sigma\, T \sqrt{\left| \det \| g_{ab} - \epsilon \frac{1}{T^2}\partial_a u \partial_b u \| \right|} \quad , \quad \epsilon = \pm 1 \, . \tag{1}$$

Here and below the following notations are used:

- g_{ab} is the induced metric on the world-volume:
$$g_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \, , \tag{2}$$
which becomes *singular* on-shell (manifestation of the lightlike nature, cf. Eq.(6) below).
- $X^\mu(\sigma)$ are the p-brane embedding coordinates in the bulk D-dimensional space-time with Riemannian metric $G_{\mu\nu}(X)$ $(\mu,\nu = 0,1,\ldots,D-1)$; $(\sigma) \equiv \left(\sigma^0 \equiv \tau, \sigma^i\right)$ with $i = 1,\ldots,p$; $\partial_a \equiv \frac{\partial}{\partial\sigma^a}$.
- u is auxiliary world-volume scalar field defining the lightlike direction (see Eq.(6) below); the choice of the sign of ϵ in (1) does not have physical effect because of the non-propagating nature of the u-field (see Appendix).
- T is *dynamical (variable)* brane tension (also a non-propagating degree of freedom, cf. Appendix).

The corresponding equations of motion w.r.t. X^μ, u and T read accordingly (with $\Gamma^\mu_{\lambda\nu}$ – Christoffel connection for the bulk metric):

$$\partial_a \left(T\sqrt{|\widetilde{g}|}\widetilde{g}^{ab}\partial_b X^\mu \right) + T\sqrt{|\widetilde{g}|}\widetilde{g}^{ab}\partial_a X^\lambda \partial_b X^\nu \Gamma^\mu_{\lambda\nu} = 0 \, , \tag{3}$$

$$\partial_a \left(\frac{1}{T}\sqrt{|\widetilde{g}|}\widetilde{g}^{ab}\partial_b u \right) = 0 \quad , \quad T^2 + \epsilon \widetilde{g}^{ab}\partial_a u \partial_b u = 0 \, , \tag{4}$$

where we have introduced the convenient notations:

$$\widetilde{g}_{ab} = g_{ab} - \epsilon\frac{1}{T^2}\partial_a u \partial_b u \quad , \quad \widetilde{g} \equiv \det \|\widetilde{g}_{ab}\| \, , \tag{5}$$

and $\widetilde{g}^{ab}$ is the inverse matrix w.r.t. $\widetilde{g}_{ab}$.

From the definition (5) and second Eq.(4) one easily finds that the induced metric on the world-volume is singular on-shell:

$$g_{ab}\left(\widetilde{g}^{bc}\partial_c u\right) = 0 \tag{6}$$

exhibiting the lightlike nature of the p-brane described by (1).

Similarly to the ordinary bosonic p-brane we can rewrite the Nambu-Goto-type action for the *LL-brane* (1) in a Polyakov-like form by employing an *intrinsic* Riemannian world-volume metric γ_{ab}:

$$S_{\mathrm{LL-Pol}} = -\frac{1}{2}\int d^{p+1}\sigma\, T b_0^{\frac{p-1}{2}}\sqrt{-\gamma}\left[\gamma^{ab}\left(g_{ab} - \epsilon\frac{1}{T^2}\partial_a u\partial_b u\right) - \epsilon b_0(p-1)\right] , \tag{7}$$

where b_0 is a positive constant. The world-volume action (7) produces the same equations of motion (3)–(4) together with the relation:

$$\gamma_{ab} = \frac{\epsilon}{b_0}\widetilde{g}_{ab} \; . \tag{8}$$

In particular, relation (8) reveals the meaning of b_0 as (inverse) proportionality factor between the intrinsic world-volume metric and the “extended” induced metric (5).

Remark. Let us note that consistency between the Lorentz nature of the intrinsic world-volume metric γ_{ab} and the Lorentz nature of the embedding space-time metric $G_{\mu\nu}$, taking into account (8), requires to set $\epsilon = 1$ in the Polyakov-type action (7).

As shown in our previous papers [7, 8, 9], using the above world-volume Lagrangian framework one can add in a natural way couplings of the *LL-brane* to bulk space-time Maxwell $\mathcal{A}_\mu$ and Kalb-Ramond $\mathcal{A}_{\mu_1\ldots\mu_{D-1}}$ gauge fields (the latter – in the case of codimension one *LL-branes*, *i.e.*, for $D = (p+1)+1$). For the Nambu-Goto-type action (1) these couplings read (second ref.[19]):

$$\widetilde{S}_{\mathrm{LL}}[q,\beta] = -\int d^{p+1}\sigma\, T\sqrt{\Big|\det\|g_{ab} - \frac{1}{T^2}(\partial_a u + q\mathcal{A}_a)(\partial_b u + q\mathcal{A}_b)\|\Big|}$$
$$-\frac{\beta}{(p+1)!}\int d^{p+1}\sigma\,\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}X^{\mu_1}\ldots\partial_{a_{p+1}}X^{\mu_{p+1}}\mathcal{A}_{\mu_1\ldots\mu_{p+1}} \tag{9}$$

with g_{ab} denoting the induced metric on the world-volume (2) and $\mathcal{A}_a \equiv \partial_a X^\mu \mathcal{A}_\mu$. Using the short-hand notation generalizing (5):

$$\bar{g}_{ab} \equiv g_{ab} - \epsilon\frac{1}{T^2}(\partial_a u + q\mathcal{A}_a)(\partial_b u + q\mathcal{A}_b) \quad , \quad \mathcal{A}_a \equiv \partial_a X^\mu \mathcal{A}_\mu \; , \tag{10}$$

the equations of motion w.r.t. X^μ, u and T acquire the form:

$$\partial_a \left(T\sqrt{|\bar{g}|}\bar{g}^{ab}\partial_b X^\mu\right) + T\sqrt{|\bar{g}|}\bar{g}^{ab}\partial_a X^\lambda \partial_b X^\nu \Gamma^\mu_{\lambda\nu} + \epsilon \frac{q}{T}\sqrt{|\bar{g}|}\bar{g}^{ab}\partial_a X^\nu (\partial_b u + q\mathcal{A}_b)\mathcal{F}^{\lambda\nu}G^{\mu\lambda} - \frac{\beta}{(p+1)!}\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}X^{\mu_1}\ldots\partial_{a_{p+1}}X^{\mu_{p+1}}\mathcal{F}_{\lambda\mu_1\ldots\mu_{p+1}}G^{\lambda\mu} = 0 \;, \quad (11)$$

with

$$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu \quad , \quad \mathcal{F}_{\mu_1\ldots\mu_D} = D\partial_{[\mu_1}\mathcal{A}_{\mu_2\ldots\mu_D]} = \mathcal{F}\sqrt{-G}\varepsilon_{\mu_1\ldots\mu_D} \quad (12)$$

being the field-strengths of the electromagnetic $\mathcal{A}_\mu$ and Kalb-Ramond $\mathcal{A}_{\mu_1\ldots\mu_{D-1}}$ gauge potentials [20], and

$$\partial_a \left(\frac{1}{T}\sqrt{|\bar{g}|}\bar{g}^{ab}(\partial_b u + q\mathcal{A}_b)\right) = 0 \;\;,\;\; T^2 + \epsilon \bar{g}^{ab}(\partial_a u + q\mathcal{A}_a)(\partial_b u + q\mathcal{A}_b) = 0 \;. \quad (13)$$

The on-shell singularity of the induced metric g_{ab} (2), *i.e.*, the lightlike property, now reads (using notation (10), cf. Eq.(6)):

$$g_{ab}\left(\bar{g}^{bc}(\partial_c u + q\mathcal{A}_c)\right) = 0 \;. \quad (14)$$

The Polyakov-type form of the world-volume action (9) becomes (using short-hand notation (10)):

$$\widetilde{S}_{\mathrm{LL-Pol}}[q,\beta] = -\frac{1}{2}\int d^{p+1}\sigma\, T b_0^{\frac{p-1}{2}}\sqrt{-\gamma}\left[\gamma^{ab}\bar{g}_{ab} - \epsilon b_0 (p-1)\right] - \frac{\beta}{(p+1)!}\int d^{p+1}\sigma\, \varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}X^{\mu_1}\ldots\partial_{a_{p+1}}X^{\mu_{p+1}}\mathcal{A}_{\mu_1\ldots\mu_{p+1}} \;, \quad (15)$$

yielding the same set of equations of motion (11)–(13) plus the counterpart of (8):

$$\gamma_{ab} = \frac{\epsilon}{b_0}\bar{g}_{ab} \quad (16)$$

with $\bar{g}_{ab}$ as in (10). Here again the above remark after Eq.(8) applies, *i.e.*, that for consistency we must set $\epsilon = 1$ within the Polyakov-type action (15).

3. Bulk Gravity/Gauge-Field System Self-Consistently Interacting With Lightlike Branes

3.1. Lagrangian Formulation

Let us now consider self-consistent bulk Einstein-Maxwell-Kalb-Ramond system coupled to $N \geq 1$ distantly separated charged codimension-one *lightlike* p-branes (in this case $D = (p+1)+1$). The pertinent Lagrangian action reads:

$$S = \int d^D x \sqrt{-G} \left[\frac{R(G)}{16\pi} - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{D!2}\mathcal{F}_{\mu_1\ldots\mu_D}\mathcal{F}^{\mu_1\ldots\mu_D} \right] + \sum_{k=1}^{N} \widetilde{S}_{\rm LL}[q^{(k)}, \beta^{(k)}] \; , \tag{17}$$

where again $\mathcal{F}_{\mu\nu}$ and $\mathcal{F}_{\mu_1\ldots\mu_D}$ are the Maxwell and Kalb-Ramond field-strengths (12) and $\widetilde{S}_{\rm LL}[q^{(k)}, \beta^{(k)}]$ indicates the world-volume action of the k-th *LL-brane* of the form (9) (or (15)).

The corresponding equations of motion are as follows:

(a) Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R = 8\pi \left(T^{(EM)}_{\mu\nu} + T^{(KR)}_{\mu\nu} + \sum_{k=1}^{N} T^{(brane-k)}_{\mu\nu} \right) \; . \tag{18}$$

The energy-momentum tensors of bulk gauge fields are given by:

$$T^{(EM)}_{\mu\nu} = \mathcal{F}_{\mu\kappa}\mathcal{F}^{\mu\nu} - G_{\mu\nu}\frac{1}{4}\mathcal{F}_{\kappa\lambda}\mathcal{F}^{\kappa\lambda} \quad , \quad T^{(KR)}_{\mu\nu} = -\frac{1}{2}\mathcal{F}^2 G_{\mu\nu} \; , \tag{19}$$

where the last relation indicates that $\Lambda \equiv 4\pi\mathcal{F}^2$ can be interpreted as dynamically generated cosmological "constant". The energy-momentum (stress-energy) tensor of k-th *LL-brane* is straightforwardly derived from the pertinent *LL-brane* action (9):

$$T^{\mu\nu}_{(brane-k)} = - \int d^{p+1}\sigma \, \frac{\delta^{(D)}\Big(x - X_{(k)}(\sigma)\Big)}{\sqrt{-G}} \, T^{(k)} \sqrt{|\bar{g}_{(k)}|}\bar{g}^{ab}_{(k)}\partial_a X^\mu_{(k)}\partial_b X^\nu_{(k)} \; , \tag{20}$$

where for each k-th *LL-brane*:

$$\bar{g}^{(k)}_{ab} \equiv g^{(k)}_{ab} - \epsilon^{(k)}\frac{1}{T^2_{(k)}}(\partial_a u^{(k)} + q^{(k)}\mathcal{A}^{(k)}_a)(\partial_b u^{(k)} + q^{(k)}\mathcal{A}^{(k)}_b)$$
$$g^{(k)}_{ab} = \partial_a X^\mu_{(k)} G_{\mu\nu}\partial_b X^\nu_{(k)} \quad , \quad \epsilon^{(k)} = \pm 1 \quad , \quad \mathcal{A}^{(k)}_a \equiv \partial_a X^\mu_{(k)}\mathcal{A}_\mu \; . \tag{21}$$

(b) Maxwell equations:

$$\partial_\nu \Big(\sqrt{-G}\mathcal{F}^{\mu\nu}\Big) - \sum_{k=1}^{N} q^{(k)} \int d^{p+1}\sigma \, \delta^{(D)}\Big(x - X_{(k)}(\sigma)\Big) \times \sqrt{|\bar{g}_{(k)}|}\bar{g}^{ab}_{(k)}\partial_a X^\mu_{(k)} \frac{\partial_b u^{(k)} + q^{(k)}\mathcal{A}^{(k)}_b}{T^{(k)}} = 0 \; , \tag{22}$$

using notations (21).

(c) Kalb-Ramond equations of motion (recall definition of $\mathcal{F}$ in (12)):

$$\varepsilon^{\nu\mu_1\ldots\mu_{p+1}}\partial_\nu\mathcal{F} - \sum_{k=1}^{N}\beta^{(k)}\int d^{p+1}\sigma\,\delta^{(D)}(x - X_{(k)}(\sigma))$$
$$\times\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}X^{\mu_1}_{(k)}\ldots\partial_{a_{p+1}}X^{\mu_{p+1}}_{(k)} = 0\,. \tag{23}$$

(d) The *LL-brane* equations of motion have already been written down in (11)–(13) above.

3.2. LL-Brane Dynamics in Static "Spherically Symmetric" Backgrounds

We will be interested in static "spherically-symmetric"-type solutions of Einstein-Maxwell-Kalb-Ramond equations with the following generic form of the bulk Riemannian metric:

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + C(\eta)h_{ij}(\theta)d\theta^i d\theta^j\,, \tag{24}$$

or, in Eddington-Finkelstein coordinates ($dt = dv - \frac{d\eta}{A(\eta)}$) :

$$ds^2 = -A(\eta)dv^2 + 2dv\,d\eta + C(\eta)h_{ij}(\theta)d\theta^i d\theta^j\,. \tag{25}$$

Here h_{ij} indicates the standard metric on p-dimensional sphere, cylinder, torus or flat Euclidean section. The "radial-like" coordinate η will vary in general from $-\infty$ to $+\infty$.

We will consider the simplest ansatz for the *LL-brane* embedding coordinates:

$$X^0 \equiv v = \tau \quad , \quad X^1 \equiv \eta = \eta(\tau) \quad , \quad X^i \equiv \theta^i = \sigma^i \;\; (i = 1,\ldots,p)\,. \tag{26}$$

Furthermore, we will use explicit world-volume reparametrization invariance of the *LL-brane* actions ((7) and (15)) to introduce the standard synchronous gauge-fixing conditions for the intrinsic world-volume metric:

$$\gamma^{00} = -1 \quad , \quad \gamma^{0i} = 0 \;\; (i = 1,\ldots,p)\,. \tag{27}$$

The latter together with second Eq.(13) and (16) (and accounting for the definition (10)) implies for the 00-component of the induced metric (2) on the *LL-brane* world-volume:

$$g_{00} \equiv \dot{X}^\mu G_{\mu\nu}\dot{X}^\nu = \frac{b_0}{T^2}\bar{g}^{ij}\left(\partial_i u + \mathcal{A}_i\right)\left(\partial_j u + \mathcal{A}_j\right) \geq 0 \tag{28}$$

which must match the condition $g_{00} \leq 0$ required by consistency between the Lorentz form of the bulk space-time metric and the Lorentz form of the *LL-brane* world-volume metric. Hence we are led to impose the ansatz:

$$\partial_i u + \mathcal{A}_i = 0 \tag{29}$$

which is consistent for static spherically symmetric bulk space-time Maxwell field $\mathcal{A}_\mu$ and whose physical meaning is that the lightlike direction for the induced metric in Eq.(14) (or Eq.(6) for electrically neutral *LL-brane*) coincides with the brane proper-time τ-direction on the world-volume.

Thus, taking into account (27) and (29), the *LL-brane* equations of motion (13) (or, equivalently, (14)) reduce to:

$$g_{00} \equiv \dot{X}^\mu G_{\mu\nu} \dot{X}^\nu = 0 \quad , \quad g_{0i} \equiv \dot{X}^\mu G_{\mu\nu} \partial_i X^\nu = 0 \; , \tag{30}$$

$$T^2 = \frac{1}{b_0} \left(\partial_0 u + \mathcal{A}_0\right)^2 \; , \; \partial_i T = 0 \; , \; \partial_0 g^{(p)} = 0 \quad \left(g^{(p)} \equiv \det \|g_{ij}\|\right) \; , \tag{31}$$

with g_{ij} being the spacelike part of the induced metric (2). Eqs.(30)–(31) with *LL-brane* embedding (26) and metric of the form (25) imply:

$$-A(\eta) + 2\,\dot{\eta} = 0 \quad , \quad \partial_\tau C = \dot{\eta}\; \partial_\eta C \,|_{\eta=\eta(\tau)} = 0 \; . \tag{32}$$

Here we will distinguish two cases. First, let us consider the case of $C(\eta)$ as non-trivial function of η (*i.e.*, proper spherically-symmetric-type space-time). In this case Eqs.(32) imply:

$$\dot{\eta} = 0 \;\to\; \eta(\tau) = \eta_0 = \mathrm{const} \quad , \quad A(\eta_0) = 0 \; . \tag{33}$$

Eq.(33) tells us that consistency of *LL-brane* dynamics in a proper spherically-symmetric-type gravitational background of codimension one requires the latter to possess a horizon (at some $\eta = \eta_0$), which is automatically occupied by the *LL-brane* ("horizon straddling" according to the terminology of Ref.[4]). Similar property – "horizon straddling", has been found also for *LL-branes* moving in rotating axially symmetric (Kerr or Kerr-Newman) and rotating cylindrically symmetric black hole backgrounds [9, 10].

Next, consider the case $C(\eta) = \mathrm{const}$ in (25), *i.e.*, the corresponding space-time manifold is of product type $\Sigma_2 \times S^p$. A physically relevant example is the Bertotti-Robinson [21, 22] space-time in $D = 4$ (*i.e.*, $p = 2$) describing Anti-de-Sitter$_2 \times S^2$ with metric (in Eddington-Finkelstein coordinates):

$$ds^2 = -\frac{\eta^2}{r_0^2} dv^2 + 2 dv d\eta + r_0^2 \left[d\theta^2 + \sin^2\theta d\varphi^2\right] \; . \tag{34}$$

At $\eta = 0$ the Bertotti-Robinson metric (34) possesses a horizon. Further, we will consider the case of Bertotti-Robinson universe with constant electric field $\mathcal{F}_{v\eta} = \pm \frac{1}{2 r_0 \sqrt{\pi}}$. In the present case the second Eq.(32) is trivially satisfied whereas the first one yields: $\eta(\tau) = \eta(0)\left(1 - \tau\frac{\eta(0)}{2r_0^2}\right)^{-1}$. In particular, if the *LL-brane* is initially (at $\tau = 0$) located on the Bertotti-Robinson horizon $\eta = 0$, it will stay there permanently. It is this particular solution which we will consider in what follows.

4. Self-Consistent Wormhole-Like Solutions with LL-Branes – General Scheme

We will construct self-consistent static "spherically symmetric" solutions of the system of Einstein-Maxwell-Kalb-Ramond equations (18)–(23) and *LL-brane* Eqs.(11)–(13) following the steps:

(i) The bulk space-time metric will be of the form:

$$ds^2 = -A(\eta)dv^2 + 2dv\,d\eta + C(\eta)h_{ij}(\theta)d\theta^i d\theta^j \;,$$
$$A(\eta_0^{(k)}) = 0 \;\; (k = 1,\ldots,N) \quad , \quad A(\eta) > 0 \;\text{ for all } \eta \neq \eta_0^{(k)} \tag{35}$$

Each horizon at $\eta = \eta_0^{(k)}$ is automatically occupied by (one of the) *LL-brane(s)* according to the *LL-brane* dynamics ("horizon straddling", cf.(32)–(33)).

(ii) Choose "vacuum" solutions of Einstein-Maxwell-Kalb-Ramond equations (18)–(23) (*i.e.*, without the delta-function terms due to the *LL-branes*) in each region $-\infty < \eta < \eta_0^{(1)}$, $\eta_0^{(1)} < \eta < \eta_0^{(2)}, \ldots, \eta_0^{(N)} < \eta < \infty$.

(iii) Match the discontinuities across each horizon at $\eta = \eta_0^{(k)}$ of the derivatives of the bulk metric, Maxwell and Kalb-Ramond field strengths using the explicit expressions for the *LL-brane* stress-energy tensors, electric and Kalb-Ramond currents systematically derived from the underlying *LL-brane* world-volume actions (15).

In particular, for the stress-energy tensor of each k-th *LL-brane* we obtain (here we suppress the index (k)):

$$T^{\mu\nu}_{(brane)} = S^{\mu\nu}\,\delta(\eta - \eta_0) \tag{36}$$

with surface energy-momentum tensor:

$$S^{\mu\nu} \equiv \frac{T}{\epsilon b_0^{1/2}}\left(\partial_\tau X^\mu \partial_\tau X^\nu - \epsilon b_0 G^{ij}\partial_i X^\mu \partial_j X^\nu\right)_{v=\tau,\,\eta=\eta_0,\,\theta^i=\sigma^i} \;, \tag{37}$$

where $G_{ij} = C(\eta)h_{ij}(\theta)$ (cf. (25)). For the non-zero components of (37) (with lower indices) and its trace we find:

$$S_{\eta\eta} = \epsilon\frac{T}{b_0^{1/2}} \quad , \quad S_{ij} = -Tb_0^{1/2}G_{ij} \quad , \quad S^\lambda_\lambda = -pTb_0^{1/2} \,. \tag{38}$$

Taking into account (36)–(38) Einstein equations (18) yield:

$$\left[\partial_\eta A\right]_{\eta_0^{(k)}} = -16\pi T^{(k)}\sqrt{b_0^{(k)}} \quad , \quad \left[\partial_\eta \ln C\right]_{\eta_0^{(k)}} = -\frac{16\pi}{p\sqrt{b_0^{(k)}}}T^{(k)} \tag{39}$$

with notation $\left[Y\right]_{\eta_0} \equiv Y\mid_{\eta\to\eta_0+0} - Y\mid_{\eta\to\eta_0-0}$ for any quantity Y.

Maxwell and Kalb-Ramond equations yield:

$$\left[\mathcal{F}_{v\eta}\right]_{\eta_0^{(k)}} = q^{(k)} \quad , \quad \left[\mathcal{F}\right]_{\eta_0^{(k)}} = -\beta^{(k)} \tag{40}$$

In Eqs.(39)–(40) $\left(T^{(k)}, b_0^{(k)}\right)$ indicate the dynamical tension and b_0 parameter of the k-th *LL-brane* occupying horizon $\eta_0^{(k)}$, with electric charge surface density $q^{(k)}$ and Kalb-Ramond coupling $\beta^{(k)}$. The second relation in (40) gives the jump of the dynamically generated cosmological constant $\Lambda \equiv 4\pi\mathcal{F}^2$ across the k-th *LL-brane*.

The only non-trivial contribution of *LL-brane* equations of motion comes from the X^0-equation which yields:

$$\partial_0 T^{(k)} + T^{(k)}\frac{1}{2}\left(\left\langle\partial_\eta A\right\rangle_{\eta_0^{(k)}} + p b_0^{(k)}\left\langle\partial_\eta \ln C\right\rangle_{\eta_0^{(k)}}\right)$$
$$-\sqrt{b_0^{(k)}}\left(q^{(k)}\left\langle\mathcal{F}_{v\eta}\right\rangle_{\eta_0^{(k)}} - \beta^{(k)}\left\langle\mathcal{F}\right\rangle_{\eta=_0^{(k)}}\right) = 0 \tag{41}$$

with notation $\langle Y\rangle_{\eta_0} \equiv \frac{1}{2}\left(Y\mid_{\eta\to\eta_0+0} + Y\mid_{\eta\to\eta_0-0}\right)$.

In what follows we will take time-independent dynamical *LL-brane* tension(s) ($\partial_0 T^{(k)} = 0$) because of matching static bulk space-time geometries. Let us also note that the appearance of mean values of the corresponding quantities with discontinuities across the horizons follows the resolution of the discontinuity problem given in [3] (see also [23]).

The wormhole-like solutions presented in the next Section share the following important properties:

(a) The *LL-branes* at the wormhole "throats" represent "exotic" matter – $T \leq 0$, *i.e.*, negative or zero brane tension implying violation of null-energy conditions as predicted by general wormhole arguments [16] (although the latter could be remedied via quantum fluctuations).

(b) The wormhole-like space-times constructed via *LL-branes* at their "throats" are *not* traversable w.r.t. the "laboratory" time of a static observer in either of the different "universes" comprising the pertinent wormhole space-time manifold. On the other hand, they *are traversable* w.r.t. the *proper time* of a traveling observer.

Proper-time traversability can be easily seen by considering dynamics of test particle of mass m_0 ("traveling observer") in a wormhole background, which is described by the world-line action:

$$S_{\rm particle} = \frac{1}{2}\int d\lambda \Big[\frac{1}{e}\,\dot{x}^\mu \dot{x}^\nu\, G_{\mu\nu} - e m_0^2\Big] \; . \tag{42}$$

Using energy $\mathcal{E}$ and orbital momentum $\mathcal{J}$ conservation and introducing the *proper* world-line time s ($\frac{ds}{d\lambda} = e m_0$), the "mass-shell" equation (the equation w.r.t. the "einbein" e produced by the action (42)) yields:

$$\left(\frac{d\eta}{ds}\right)^2 + \mathcal{V}_{\rm eff}(\eta) = \frac{\mathcal{E}^2}{m_0^2} \quad , \quad \mathcal{V}_{\rm eff}(\eta) \equiv A(\eta)\Big(1 + \frac{\mathcal{J}^2}{m_0^2 C(\eta)}\Big) \tag{43}$$

where the metric coefficients $\mathcal{A}(\eta)$, $C(\eta)$ are those in (35). Irrespectively of the specific form of the "effective potential" in (43), a "radially" moving (with zero "impact" parameter $\mathcal{J} = 0$) traveling observer (and with sufficiently large energy $\mathcal{E}$) will always cross within finite amount of proper-time through any "throat" ($\eta = \eta_0^{(k)}$) from one "universe" to another and possibly even shuttle between them (cf. Subsection 5.4 below).

5. Examples

Henceforth we will use the following acronyms for brevity: "BR"="Bertotti-Robinson", "Schw"="Schwarzschild", "RN"= "Reissner-Nordström", "(A)dS"="(Anti-)de-Sitter", "SdS" = "Schwarzschild-de-Sitter", and *LL-brane* matching will be denoted by "|".

5.1. Symmetric Wormhole with Reissner-Nordström Geometry

It consists of two identical copies of exterior RN region ($r > r_0$, r_0 denoting the *outer* RN horizon) – "left" RN "universe" ($\eta < 0$) and "right" RN "universe" ($\eta > 0$) glued together via a LL-brane sitting on $r = r_0$ ($\eta = 0$):

$$ds^2 = -A(\eta)dv^2 + 2dv\,d\eta + C(\eta)\left[d\theta^2 + \sin^2\theta\, d\varphi^2\right] \; , \tag{44}$$

$$A(\eta) = 1 - \frac{2m}{r_0 + |\eta|} + \frac{Q^2}{(r_0+|\eta|)^2} \quad , \quad C(\eta) = (r_0 + |\eta|)^2 \; , \tag{45}$$

$$A(0) = 0 \quad , \quad A(\eta) > 0 \;\text{ for } \eta \neq 0 \; . \tag{46}$$

RN mass is determined by the dynamical LL-brane tension T:

$$(16\pi\,|T|\sqrt{b_0}\,m - 1)\left(m^2 - Q^2\right) + 16\pi^2 T^2 b_0 Q^4 = 0 \; . \tag{47}$$

In the particular case of Schwarzschild wormhole (Einstein-Rosen "bridge", $Q = 0$): $m = 1/8\pi|T|$.

5.2. Non-singular Black Hole

It is described by the metric:

$$ds^2 = -A(r)dv^2 + 2dv\,dr + r^2\left[d\theta^2 + \sin^2\theta\,d\varphi^2\right] \; ; \tag{48}$$

$$A(r) \equiv A_{(-)}(r) = 1 - Kr^2 \quad , \text{ for } r < r_0 \quad \text{(de Sitter)} \; , \tag{49}$$

$$A(r) \equiv A_{(+)}(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} \quad , \text{ for } r > r_0 \quad \text{(RN)} \; , \tag{50}$$

where r_0 is the common horizon $A_{(\pm)}(r_0) = 0$, $r_0 = m - \sqrt{m^2 - Q^2}$ (internal RN).

An electrically charged LL-brane occupies the horizon $r = r_0$ and uniquely determines all parameters $r_0 = \frac{1}{\sqrt{K}}$, $m = \frac{2}{\sqrt{K}}$, $Q^2 = \frac{3}{K}$, with $\Lambda = 3K = \frac{4\pi}{3}\beta^2$ – *dynamically generated* cosmological const in the interior de-Sitter region through the Kalb-Ramond LL-charge β. Apparently there is *no* black hole singularity at $r = 0$.

5.3. Asymmetric Wormhole – Schw-dS | RN

The overall metric is $ds^2 = -A(\eta)dv^2 + 2dv\,d\eta + (r_0 + |\eta|)^2\left[d\theta^2 + \sin^2\theta\,d\varphi^2\right]$ with $A(0) = 0$. Here we have:

(i) "left universe" – exterior region of Schwarzschild-de-Sitter space-time above the *inner* (Schwarzschild-type) horizon r_0:

$$A(\eta) = 1 - \frac{2m_1}{r_0 - \eta} - K(r_0 - \eta)^2 \quad \text{for } \eta < 0 \; ; \tag{51}$$

(ii) "right universe" – exterior Reissner-Nordström region beyond the *outer* RN horizon r_0:

$$A(\eta) = 1 - \frac{2m_2}{r_0 + \eta} + \frac{Q^2}{(r_0 + \eta)^2} \quad \text{for } \eta > 0 \; . \tag{52}$$

Charged LL-brane occupies the common horizon (wormhole "throat") and determines all wormhole parameters via its charges (q, β):

$$m_1 = \frac{\sqrt{b_0}}{4\pi|T|}\Bigl(1 - \frac{b_0\beta^2}{3\pi T^2}\Bigr) \quad , \quad m_2 = \frac{\sqrt{b_0}}{4\pi|T|}\Bigl(1 + \frac{4q^2}{\pi T^2}\Bigr) \; , \tag{53}$$

$$r_0 = \frac{\sqrt{b_0}}{2\pi|T|} \quad , \quad T^2 = \frac{\beta^2 + 4q^2}{2\pi(1 - 4b_0)} \quad , \quad Q^2 = \frac{16\pi}{b_0}q^2 r_0^4 \; . \tag{54}$$

including the dynamically generated cosmological const $\Lambda = 3K = 4\pi\beta^2$ in the "left" universe.

5.4. Compactification/Decompactification Transitions

These are wormhole-like solution with two widely separated LL-branes sitting at horizons $\eta = \eta_0 \equiv 0$ and $\eta = \bar{\eta}_0$, with metric:

$$ds^2 = -A(\eta)dv^2 + 2dvd\eta + C(\eta)\left[d\theta^2 + \sin^2\theta d\varphi^2\right] , \quad (55)$$

$$A(0) = 0 \; , \; A(\bar{\eta}_0) = 0 \; , \; \bar{\eta}_0 \equiv \bar{r}_0 - r_0 > 0 \; , \; A(\eta) > 0 \text{ for } \eta \neq 0, \bar{\eta}_0 \; , \quad (56)$$

describing *three* pairwise matched space-time regions:

(i) "left" Bertotti-Robinson "universe" ($AdS_2 \times S^2$) for $\eta < 0$ where:

$$A(\eta) = \frac{\eta^2}{r_0^2} \quad , \quad C(\eta) = r_0^2 \quad , \quad \mathcal{F}_{v\eta} = \pm\frac{1}{2\sqrt{\pi}\, r_0} \; ; \qquad (57)$$

(ii) "middle" Reissner-Nordström-de-Sitter "universe" for $0 < \eta < \bar{r}_0 - r_0$ with:

$$A(\eta) = 1 - \frac{2m}{r_0 + \eta} + \frac{Q^2}{(r_0 + \eta)^2} - \frac{4\pi\beta^2}{3}(r_0 + \eta)^2 \; , \qquad (58)$$

$$C(\eta) = (r_0 + \eta)^2 \quad , \quad \mathcal{F}_{v\eta} = \frac{Q}{\sqrt{4\pi}(r_0 + \eta)^2} \quad , \qquad (59)$$

where r_0 and $\bar{r}_0$ ($\bar{r}_0 > r_0$) are the intermediate (outer RN) and the outmost (de-Sitter) horizons of the standard RN-de-Sitter space-time (note the dynamically generated cosmological const $\Lambda = 4\pi\beta^2$ in (58));

(iii) another "right" Bertotti-Robinson "universe" ($AdS_2 \times S^2$) for $\eta > \bar{r}_0 - r_0$:

$$A(\eta) = \frac{(\eta - \bar{r}_0 + r_0)^2}{\bar{r}_0^2} \quad , \quad C(\eta) = \bar{r}_0^2 \quad , \quad \mathcal{F}_{v\eta} = \pm\frac{1}{2\sqrt{\pi}\, \bar{r}_0} \; . \qquad (60)$$

Traveling observer along η-direction will "shuttle" between the three "universes" crossing consecutively both LL-branes at the "throats" within *finite* intervals of his/her proper time.

5.5. Multi-"throat" wormhole Schw | SdS | SdS | Schw

This is a wormhole-like solution with metric:

$$ds^2 = -A(\eta)dv^2 + 2dvd\eta + (r_0 + \eta)^2\left[d\theta^2 + \sin^2\theta d\varphi^2\right]$$
$$A(0) = 0 \quad , \quad A(\pm(\bar{r}_0 - r_0)) = 0$$

describing *four* pairwise matched space-time regions via 3 widely separated LL-branes located at $\eta = 0$ and $\eta = \pm(\bar{r}_0 - r_0)$:

(i) "left-most" ($\eta < -(\bar{r}_0 - r_0)$) and "right-most" ($\eta > \bar{r}_0 - r_0$) "universes" comprising the exterior Schwarzschild region beyond the Schwarzschild horizon at $\bar{r}_0$:

$$A(\eta) = 1 - \frac{\bar{r}_0}{r_0 + |\eta|} \quad \text{for } |\eta| > \bar{r}_0 - r_0 \;, \tag{61}$$

(ii) two "middle" "universes", for $-(\bar{r}_0 - r_0) < \eta < 0$ and for $0 < \eta < \bar{r}_0 - r_0$ – two identical copies of the intermediate region of Schwarzschild-de-Sitter space-time between the inner (Schwarzschild) horizon at r_0 and the outer (de-Sitter) horizon at $\bar{r}_0$:

$$A(\eta) = 1 - \frac{2m}{r_0 + |\eta|} - \frac{4\pi\beta^2}{3}(r_0 + |\eta|)^2 \quad \text{for } |\eta| < \bar{r}_0 - r_0 \;, \tag{62}$$

where $A(0) = 0$ (inner SdS horizon) and $A(\pm(\bar{r}_0 - r_0)) = 0$ (outer SdS horizon) and with dynamically generated (by the LL-branes) cosmological const $\Lambda = 4\pi\beta^2$.

5.6. Lightlike Braneworld

This is a solution with a bulk $D=5$ space-time consisting of two identical copies of the exterior region of $D=5$ AdS-Schwarzschild black hole beyond the horizon r_0 ("left" universe for $\eta < 0$ and "right" universe for $\eta > 0$) glued together by a lightlike 3-brane with flat 4-dim world-volume located at the horizon ($\eta = 0$):

$$ds^2 = -A(\eta)dv^2 + 2dv\, d\eta + K(r_0 + |\eta|)^2 d\vec{x}^2 \;, \tag{63}$$

$$A(\eta) = K(r_0 + |\eta|)^2 - \frac{m}{(r_0 + |\eta|)^2} \tag{64}$$

with $A(0) = 0$ and $A(\eta) > 0$ for $\eta \neq 0$, where $\Lambda = -6K$ is the bare $D=5$ cosmological constant.

The bulk space-time parameters (K, m) are related to the LL-brane parameters (T, b_0) as: $T^2 = 3K/8\pi^2$ and $b_0 = \frac{2}{3}\sqrt{Km}$.

Because of the shape of the "effective potential" $A(\eta)$ (64) a traveling observer along the extra 5-th dimension will "shuttle" between the two "universes" crossing in either direction the $D=4$ braneworld within *finite* intervals of his/her proper time.

6. Conclusions

To conclude let us recapitulate the crucial properties of the dynamics of *LL-branes* interacting with gravity and bulk space-time gauge fields:

(i) "Horizon straddling" – automatic positioning of *LL-branes* on (one of) the horizon(s) of the bulk space-time geometry.

(ii) Intrinsic nature of the *LL-brane* tension as an additional *degree of freedom* unlike the case of standard Nambu-Goto p-branes (where it is a given *ad hoc* constant), and which might in particular acquire zero or negative values.

(iii) The stress-energy tensors of the *LL-branes* are systematically derived from the underlying *LL-brane* world-volume Lagrangian actions and provide the appropriate source terms on the r.h.s. of Einstein equations to enable the existence of consistent non-trivial wormhole-like solutions.

(iv) *LL-branes* naturally couple to Kalb-Ramond bulk space-time gauge fields which results in *dynamical* generation of space-time varying cosmological constant. In particular, the latter is responsible for creation of a non-singular black hole with de Sitter interior region below the horizon.

(v) The above properties of *LL-branes* trigger spontaneous compactification/decompactification transitions in the bulk space-time manifold.

Further explicit solutions describing multi-"throat" wormhole-like space-times of the form "BR | SdS | SdS | BR", "BR | SdS | Schw", "Cyclic" SdS, as well as "flat Minkowski | AdS-RN" will appear in a subsequent paper.

Appendix

Let us consider for simplicity the *LL-brane* Polyakov-type action (7) for $p = 0$, *i.e.*, the case of *lightlike* (*LL-*) particle:

$$S_{\mathrm{LL-particle}} = \frac{1}{2}\int d\tau T b_0^{-\frac{1}{2}} \left[\frac{1}{e}\Big(\dot{X}^2 - \epsilon\frac{\dot{u}^2}{T^2}\Big) - \epsilon b_0 e\right] , \tag{65}$$

where $\dot{X}^2 \equiv \dot{X}^\mu G_{\mu\nu} \dot{X}^\nu$ and e is the einbein ($\gamma_{00} = -e^2$, $\sqrt{-\gamma} = e$). We will show that the LL-particle (65) is dynamically equivalent to the standard *massless* particle described by the action (42) with $m_0 = 0$.

Indeed, the action (65) produces the following equations of motion w.r.t. e, T, u and X^μ:

$$\dot{X}^2 + \epsilon\Big(b_0 e^2 - \frac{\dot{u}^2}{T^2}\Big) = 0 \quad , \quad \dot{X}^2 - \epsilon\Big(b_0 e^2 - \frac{\dot{u}^2}{T^2}\Big) = 0 \, , \tag{66}$$

$$\partial_\tau\Big(\frac{\dot{u}}{eT}\Big) = 0 \quad , \quad \partial_\tau\Big(\frac{T}{e}\,\dot{X}^\mu\Big) + \frac{T}{e}\,\dot{X}^\nu \dot{X}^\lambda\, \Gamma^\mu_{\nu\lambda} = 0 \, . \tag{67}$$

Eqs.(66) imply $\dot{X}^2 = 0$ and $e^2 b_0 = \dot{u}^2/T^2$, where the first expression is the standard massless constraint following from the standard action (42) (with $m_0 = 0$) upon varying w.r.t. e, whereas the second relation makes the first Eq.(67) an identity. The last Eq.(67) is obviously equivalent to the standard geodesic equation up to a world-line τ-reparametrization.

Within the canonical Hamiltonian approach, introducing the canonical momenta (using the short-hand notation $\widetilde{e} \equiv e b_0^{1/2}$) $P_\mu = \frac{T}{\widetilde{e}} G_{\mu\nu} \dot{X}^\nu$ and $p_u \equiv -\frac{\epsilon}{\widetilde{e}T}\,\dot{u}$ we obtain the canonical Hamiltonian:

$$H_c = \frac{\widetilde{e}}{2T} P^2 - \epsilon\frac{\widetilde{e}T}{2}\left(p_u^2 - 1\right) \quad , \quad P^2 \equiv P_\mu G^{\mu\nu} P_\nu \, . \tag{68}$$

Preservation of the primary constraints $p_e = 0$ and $p_T = 0$ (vanishing canonical momenta of e and T) by (68) yields the secondary first-class constraints:

$$P^2 = 0 \quad , \quad p_u^2 - 1 = 0 \; . \tag{69}$$

Thus, we deduce that e, T, u are non-propagating "pure-gauge" degrees of freedom and we are left with the first relation (69) which is the standard canonical massless constraint resulting from the standard action (42) (with $m_0 = 0$) within the Hamiltonian formalism.

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